*Granger causality*

A time series x is said to “Granger cause” a time series y, if the past values of x provide statistically significant information to predict the next value of y (Granger, 1969). The prediction is computed using AR models. Two AR models are required: an unrestricted AR model where the history of all time series is assumed to contribute to the prediction of the current value of a time series; and a restricted AR model where the time series whose causality value (on the other time series) is computed is excluded from the history. Given two time series x and y, the unrestricted model is defined as:

(2)

While the restricted model is defined as:

(3)

The parameters of the models (i.e., the “weights of the history” , , and ) can be computed by using Ordinary Least Squares. The model order, i.e., the length *j* of the history used by the model, can be automatically computed using one of two widely used methods: the Akaike Information Criterion (Akaike, 1974) or the Bayesian Information Criterion (Schwartz, 1978). Both criteria search for a trade-off between the accuracy of the AR model in the modeling of the training data (measured, e.g., computing the residual sum squares of error of the AR model) and the number of model parameters.

Once we have computed the parameters of the models, the magnitude of the causality from x to y and from y to x can be measured respectively as:

,

where E and H are the model error variances:

, ,

Once the Granger causality values have been computed, we need to test their statistical significance, i.e., we need to infer the significant causal relations. A significance test can be done by carrying out an F-test of the null hypothesis that the model parameters referring to the time series of which we compute the “causal strength” (on the other time series) are all zero (e.g., parameters in model (2) to test the significance of ). When more than two time series are analyzed some corrections (e.g., the Bonferroni correction) are applied to the F-test.

*Conditional Granger causality*

When the interaction of more than two time series is addressed, repeated pair-wise Granger causality computations can lead to misleading results. To avoid that, a simple extension of Granger causality, sometimes referred to as Conditional Granger causality, has been proposed by (Ding et al., 2006). Suppose we have three time series x, y and z, then the Conditional Granger causality from y to x given z () is defined as the log ratio of the error variance of the restricted model where only y is excluded from the history (when modeling x) and the variance of the unrestricted model, where the history of all time series x, y and z is included.

*Musician Driving Force*

We define the driving force of musician on musician  at time as:

where is the slice of the time series of in the observation window centered at time , and

The Musician Driving Force (MDF) of a musician at time  is defined as:



where is the number of musicians in the quartet (4).

Note that although the system we analyzed (i.e., the quartet) has more than 2 variables, we used (pairwise) G-causality as we did not want to miss any actual causal relation that may not be captured, or at least not entirely captured, if conditional G-causality were used instead. Suppose we have a time series x that influences two time series y and z, whose behavior is very similar and almost entirely dependent on x. By using conditional G-causality, may not be significant since most of the behavior of z could be predicted by observing the history of y. Of course by using pairwise G-causality we can infer erroneous causal relations (e.g., in the example above, a significant ) but, by taking into account both direct and "mediated" causal relations, we should guarantee that the actual leader time series is the one that has the highest number of significant and positive F.

*Inter-musician communication*

The Inter-musician communication (IMC) in the observation window centered at time is defined as:



where is the set of all musicians (i.e., quartet) and  is G-causality of  on given .

Note that by using conditional G-causality all significant flows of information among musicians are counted only once and it does not matter whether a flow of information is mistakenly attributed to a musician pair rather that to the correct one.

*Detailson the G-causality based analysis*

G-causality and conditional G-causality were computed over the z-normalized time series of the Euclidian distances between the musician heads and the “ear” of the quartet every 500 milliseconds (i.e., 2Hz “sampling frequency”) on 3-second sliding windows using the “Granger causality connectivity analysis” Matlab toolbox (Seth, 2010).

The model order of the AR models was automatically determined using the Akaike Information Criterion.

Since the validity of the inferred causal relations depends on the validity of the AR models (more specifically on the validity of the unrestricted AR models) we tested the validity of the AR models using 3 different tests: the Durbin-Watson test, a consistency test, and a goodness of fit test (see Seth, 2010 for details). In more than the 85% of the cases (i.e., of the observation windows) the AR models passed all three tests. We observed that when we reduced the length of the observation windows the AR models became less reliable. Thus we choose the 3-second length as a trade-off between the temporal resolution of IMC and MDF (which also depends on the window shift) and their reliability.

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